

$$z = z^2 - z$$

$$\begin{aligned} a) \quad z &= (x+iy)^2 - (x+iy) \\ &= x^2 + 2ixy - y^2 - x - iy \\ &= x^2 - x - y^2 + i(2xy - y) \\ &= x^2 - y^2 - x + iy(2x - 1) \end{aligned}$$

$$b) \quad z \in \mathbb{R} \Leftrightarrow \operatorname{Im}(z) = 0$$

$$\operatorname{Im}(z) = y(2x - 1)$$

On peut choisir $z \in \mathbb{R} : z = 3; z = 42$

ou z tel que $2\operatorname{Re}(z) = 1 : z = \frac{1}{2}$
 $z = \frac{1}{2} - 3i$

$$c) \quad z \in i\mathbb{R} \Leftrightarrow \operatorname{Re}(z) = 0$$

$$\operatorname{Re}(z) = x^2 - y^2 - x$$

On peut choisir: $z = 1 + 0i$

$$z = 0 + 0i$$

$$z = -1 + i\sqrt{2}$$

$$z = -2 + i\sqrt{6}$$

$$z = -2 - i\sqrt{6}$$

37p30

$$z = z^2 - 3$$

$$\begin{aligned} a) \quad z &= (x+iy)^2 - 3 \\ &= x^2 - y^2 + 2ixy - 3 \\ &= (x^2 - y^2 - 3) + i(2xy) \end{aligned}$$

$$b) \quad z \in \mathbb{R} \implies \operatorname{Im}(z) = 0$$

$$\implies 2xy = 0$$

$$\implies x = 0 \text{ ou } y = 0$$

$$\implies z \in \mathbb{R} \text{ ou } z \in i\mathbb{R}$$

$$\begin{aligned} c) \quad (2+3i)^2 - 3 &= 4 - 9 + 12i - 3 \\ &= -8 + 12i \end{aligned}$$

Non pour $z = 2+3i$; $z = -8+12i$

$$\operatorname{Re}(z) = -8 \neq 0$$

$$z = (\bar{z} + 1 + i)^2 = (x + iy + 1 + i)^2$$

a)

$$= x^2 - y^2 + 1 - 1 + 2ixy + 2x + 2ix + 2iy - 2y + 2i$$

$$= x^2 - y^2 + 2x - 2y + i(2x + 2y + 2xy + 2)$$

b) $z \in i\mathbb{R} \Leftrightarrow \operatorname{Re}(z) = 0$

$$\Leftrightarrow x^2 - y^2 + 2x - 2y = 0$$

$$\Leftrightarrow (x - y)(x + y + 2) = 0$$

$$\Leftrightarrow \begin{cases} z = x + ix; x \in \mathbb{R} \\ \text{ou} \\ z = x + i(-x - 2); x \in \mathbb{R} \end{cases}$$

c) $\operatorname{Im} z = -2i; x = 0; y = -0 - 2 = -2$

ou $z \in i\mathbb{R}$